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Individual Assignments #58

Assignment: Section 1.6: 8 (use contradiction), 22, 24, 26, 30; Section 1.7: 6, 28, 34

# S1.6 Q8

If n is a perfect square then n+2 is not a perfect square.

P = a is perfect square

Q = n + 2 is not a perfect square

Assume for sake for sake of contradiction P →¬Q.

⟹ n+2 is a perfect square

The difference between one square and the next is calculable. Assume x2 for a perfect square and (x+1)2 for the next perfect square after x.

The difference: (x+1)2 - x2 = x2 + 2x + 1 - x2 = 2x + 1.

So, the smallest distance between perfect squares is when x = 0 ⟹ difference of 1.

The next jump is when x = 1 ⟹ difference of 3.

Thus n + 1 and n + 3 are possible. But n + 2 is not.

Proven by contradiction. ⧠

# S1.6 Q22

P = you pick 3 socks out of a drawer of only blue and black socks

Q = 2 socks are blue ⋁ 2 socks are black.

Since there are only four possible combinations of socks as shown in the table, an exhaustive proof is effective.

|  |  |  |
| --- | --- | --- |
| **Sock 1** | **Sock 2** | **Sock 3** |
| **black** | **black** | **black** |
| **black** | **black** | **blue** |
| **black** | **blue** | **blue** |
| **blue** | **blue** | **blue** |

Thus, any combination has either 2 black socks or 2 blue socks. ⧠

# S1.6 Q24

P = at least 3 of any 25 days are in the same month of the year. Assume for contradiction that 2 or less days are in any given month. Since there are 12 months in a year we are left with only 24 days chosen. This is shows we need at least three in a month because we need to choose a total of 25 days.

Proven by contradiction. ⧠

# S1.6 Q30

Show that a < b ≡ (a+b)/2 > a ≡ (a+b)/2 < b.

Case a<b:

⟹a - b < 0

Case (a+b)/2 > a:

⟹a + b – 2a > a

⟹ - (b - a > 0)

⟹a – b < 0

Case (a+b)/2 < b:

⟹a + b – 2b < 0

⟹a – b < 0

Thus all three cases evaluate to the same thing.

a - b < 0 ≡ a - b < 0 ≡ a - b < 0 ⧠

# S1.7 Q6

Prove that ∃xP(x), P: ni= n1 + n2 … ni-1

Constructive proof: 1 + 2 = 3⧠

# S1.7 Q28

P = no solutions for x & y for if x and y are integers.

First, solve for y: . When x is 0 or 1 or 2 the equation evaluates to a non integer. When x is three or greater the result is an imaginary number, hence no integer can be used for x and y to solve the equation. It also holds for the negative integers, because x2 of any negative integer is positive. ⧠

In the interest of completeness, if you do the same for x: : . When y is 0 or 1 the equation evaluates to a non integer. When y is 2 or greater the result is an imaginary number, hence no integer can be used for x and y to solve the equation. It also holds for the negative integers, because y2 of any negative integer is positive. ⧠

# S1.7 Q34

P = between every rational number and every irrational number there is an irrational number.

A rational number by definition is a quotient of two integers, . It is less than , where is some irrational number. By dividing the by two we get a smaller irrational number, because .

So is between and . This term can always be reduced by half, hence there will always be an irrational number between any rational number and irrational number.